Asynchronicity in temporal team semantic

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Asynchronous Hyperproperties: From Theory to Practice
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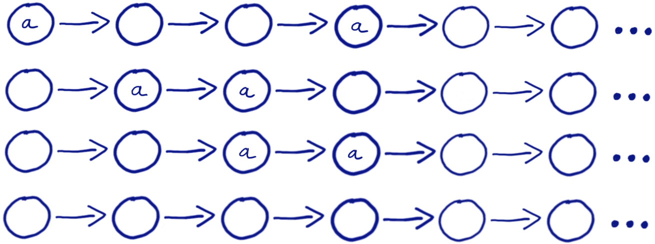
Examples: HyperLTL vs. TeamLTL

There is a timepoint (common for all traces) after which a does not occur.
Not expressible in HyperLTL, but is in HyperQPTL.

$$\exists p \forall \pi \, \mathsf{F} p \wedge \mathsf{G}(p \rightarrow \mathsf{G} \neg a_\pi)$$

Expressible in synchronous TeamLTL: $\mathsf{FG} \neg a$



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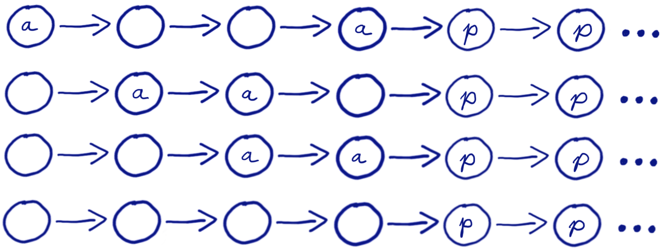

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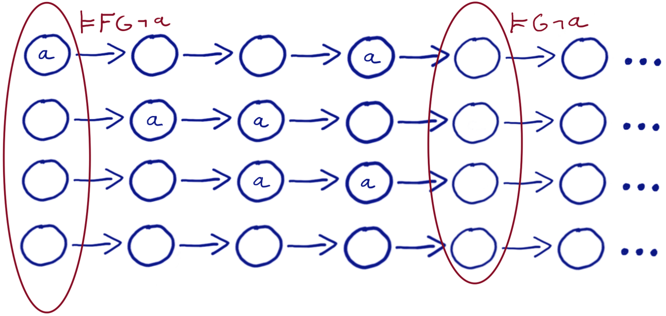

Examples: HyperLTL vs. TeamLTL

Temporal team semantics is universal and for now synchronous

$(T, i) \models p$ iff $\forall t \in T : t[i](p) = 1$ $\quad$ $(T, i) \models \neg p$ iff $\forall t \in T : t[i](p) = 0$

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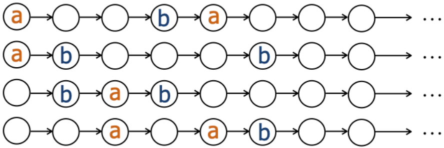
$(T, i) \models \mathsf{F}\varphi$ iff $(T, j) \models \varphi$ for some $j \geq i$ $\quad$ $(T, i) \models \mathsf{G}\varphi$ iff $(T, j) \models \varphi$ for all $j \geq i$

Examples: HyperLTL vs. TeamLTL

A trace-set T satisfies $\varphi \vee \psi$ if it decomposed to sets T_φ and T_ψ satisfying φ and ψ.

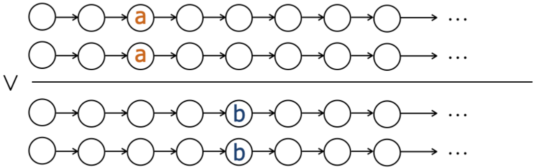
HyperLTL:

$$\forall\pi.\forall\pi'.\ \mathrm{F}((a_\pi \wedge a_{\pi'}) \vee (b_\pi \wedge b_{\pi'}))$$



TeamLTL:

$$(\mathrm{F}\ a) \vee (\mathrm{F}\ b)$$



Examples: HyperLTL vs. TeamLTL

A trace-set T satisfies $\varphi \vee \psi$ if it decomposed to sets T_φ and T_ψ satisfying φ and ψ.

$$(T, i) \models \varphi \vee \psi \text{ iff } (T_1, i) \models \varphi \text{ and } (T_2, i) \models \psi, \text{ for some } T_1 \cup T_2 = T$$
$$(T, i) \models \varphi \wedge \psi \text{ iff } (T, i) \models \varphi \text{ and } (T, i) \models \psi$$

HyperLTL:

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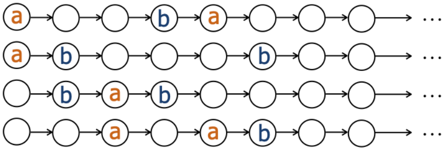

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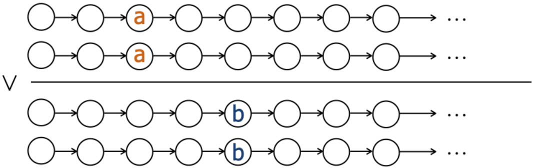


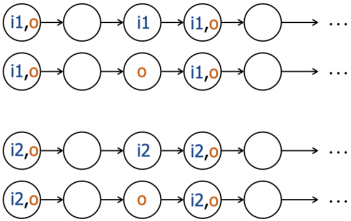
Examples: HyperLTL vs. TeamLTL

Dependence atom $\mathrm{dep}(p_1, \ldots, p_m, q)$ states that $p_1, \ldots, p_m$ functionally determine q:

TeamLTL:

$$(\mathrm{G}\ dep(i1, o)) \vee (\mathrm{G}\ dep(i2, o))$$

Nondeterministic dependence: “*o* either depends on i1 or on i2”



“whenever the traces agree on i1, they agree on o”

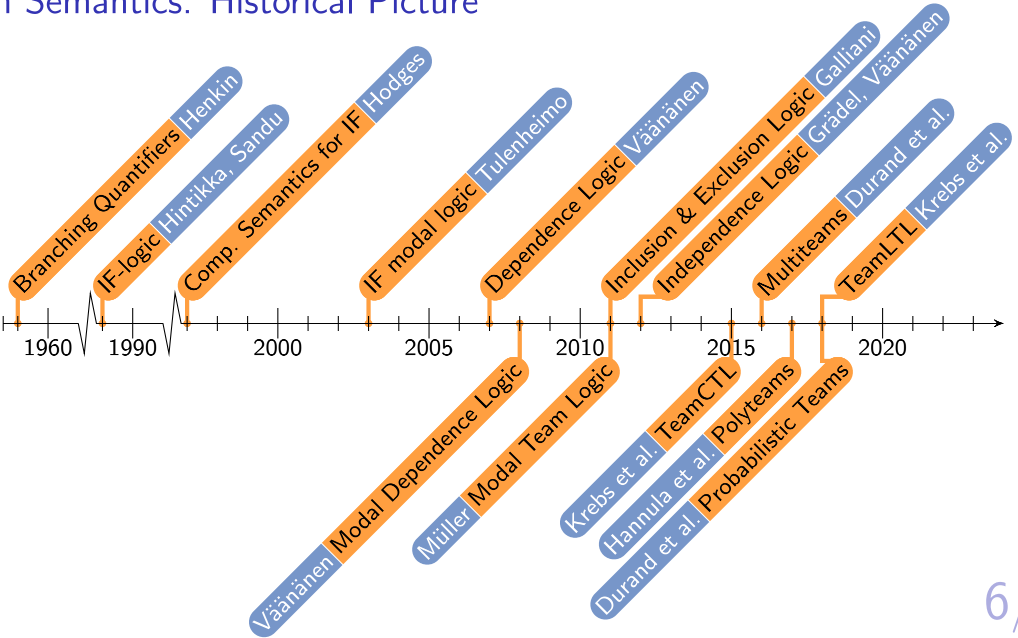
$\vee$

“whenever the traces agree on i2, they agree on o”

Core of Team Semantics

- In most studied logics formulae are evaluated in a single state of affairs.
  E.g.,
  - a first-order assignment in first-order logic,
  - a propositional assignment in propositional logic,
  - a possible world of a Kripke structure in modal logic.
- In team semantics sets of states of affairs are considered.
  E.g.,
  - a set of first-order assignments in first-order logic,
  - a set of propositional assignments in propositional logic,
  - a set of possible worlds of a Kripke structure in modal logic.
- These sets of things are called teams.

Team Semantics: Historical Picture



Logics for traceproperties and hyperproperties

Recipe for logics for hyperproperties:
A logic for traceproperties $\rightsquigarrow$ add trace quantifiers

In LTL the satisfying object is a trace: $T \models \varphi$ iff $\forall t \in T : t \models \varphi$

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \vee \varphi) \mid X\varphi \mid \varphi U \varphi$$

In HyperLTL the satisfying object is a set of traces and a trace assignment: $\Pi \models_T \varphi$

$$\varphi ::= \exists\pi\varphi \mid \forall\pi\varphi \mid \psi$$
$$\psi ::= p_\pi \mid \neg\psi \mid (\psi \vee \psi) \mid X\psi \mid \psi U \psi$$

HyperQPTL extends HyperLTL by (uniform) quantification of propositions: $\exists p\varphi$, $\forall p\varphi$

LTL, HyperLTL, and TeamLTL

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In TeamLTL the satisfying object is a set of traces. We use team semantics: $(T, i) \models \varphi$

$$\varphi ::= p \mid \neg p \mid (\varphi \vee \varphi) \mid (\varphi \wedge \varphi) \mid X\varphi \mid \varphi U \mid \varphi W \varphi$$

+ new atomic statements (dependence and inclusion atoms: $\mathrm{dep}(\vec{p}, q)$, $\vec{p} \subseteq \vec{q}$)
+ additional connectives (Boolean disjunction, contradictory negation, etc.)

Extensions are a well-defined way to delineate expressivity and complexity

Temporal team semantics

Definition

Temporal team is (T, i), where T a set of traces and $i \in \mathbb{N}$.

$$
\begin{array}{lll}
(T,i) \models p & \text{iff} & \forall t \in T : t[0](p) = 1 \\
(T,i) \models \neg p & \text{iff} & \forall t \in T : t[0](p) = 0 \\
(T,i) \models \phi \wedge \psi & \text{iff} & (T,i) \models \phi \text{ and } (T,i) \models \psi \\
(T,i) \models \phi \vee \psi & \text{iff} & (T_1,i) \models \phi \text{ and } (T_2,i) \models \psi, \text{ for some } T_1, T_2 \text{ s.t. } T_1 \cup T_2 = T \\
(T,i) \models \mathsf{X}\varphi & \text{iff} & (T,i+1) \models \varphi \\
(T,i) \models \phi \mathsf{U} \psi & \text{iff} & \exists k \geq i \text{ s.t. } (T,k) \models \psi \text{ and } \forall m : i \leq m < k \Rightarrow (T,m) \models \phi \\
(T,i) \models \phi \mathsf{W} \psi & \text{iff} & \forall k \geq i : (T,k) \models \phi \text{ or } \exists m \text{ s.t. } i \leq m \leq k \text{ and } (T,m) \models \psi
\end{array}
$$

As usual $\mathsf{F}\varphi := (\top \mathsf{U} \varphi)$ and $\mathsf{G}\varphi := (\varphi \mathsf{W} \bot)$.

$\mathrm{TeamLTL}(⩒, \subseteq)$ is the extension with the atoms and extra connectives in the brackets.

Generalised atoms and complete logics

For a tuple of LTL-formulae $(\varphi_1, \ldots, \varphi_n)$ one can imagine various statemens:

- functional dependence between truth values,
- independence of truth values, $\mathrm{user} \perp_{\mathrm{party}} \mathrm{options}$ (users etc. coded as bit strings).
- public bits do not reveal confidental bits $(o_1, \ldots, o_n, c) \subseteq (o_1, \ldots, o_n, \neg c)$.

We can implement each of such statement as an atomic formula.

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Theorem

The logic $\mathrm{TeamLTL}(\varovee, \mathrm{NE}, \overset{1}{\mathsf{A}})$ *can express all such statements.*

- $(T, i) \models \varphi \varovee \psi$ iff $(T, i) \models \varphi$ or $(T, i) \models \psi$
- $(T, i) \models \mathrm{NE}$ iff $T \neq \emptyset$.
- $(T, i) \models \overset{1}{\mathsf{A}}\varphi$ iff $(\{t\}, i) \models \varphi$, for all $t \in T$.

Complexity results for synchronous TeamLTL

Logic	Model Checking Result
$\mathrm{TeamLTL}$ without $\vee$	in PSPACE
k-coherent $\mathrm{TeamLTL}(\sim)$	in EXPSPACE
left-flat $\mathrm{TeamLTL}(\otimes, \overset{1}{\mathsf{A}})$	in EXPSPACE
$\mathrm{TeamLTL}(\subseteq, \otimes)$	Σ^0_1-hard
$\mathrm{TeamLTL}(\subseteq, \otimes, \mathsf{A})$	Σ^1_1-hard
$\mathrm{TeamLTL}(\sim)$	complete for third-order arithmetic [Luck 2020]

Table: Complexity results.

- k-coherence: $(T, i) \models \varphi$ iff $(S, i) \models \varphi$ for all $S \subseteq T$ s.t. $|S| \leq k$
- left-flatness: Restrict U and W syntactically to $(\overset{1}{\mathsf{A}}\varphi \mathsf{U} \psi)$ and $(\overset{1}{\mathsf{A}}\varphi \mathsf{W} \psi)$
- $\sim$ is contradictory negation and $\mathrm{TeamLTL}(\sim)$ subsumes all the other logics

Expressivity can be bounded by variants of HyperQPTL

$\mathrm{TeamLTL}(\varovee, \overset{1}{\mathsf{A}})$	$\leq$	$\overset{u}{\exists}{}^*_q \forall_\pi \mathrm{HyperQPTL}$ (assuming left-flatness)
	$\leq$	$\exists_p \overset{u}{Q}{}^*_p \forall_\pi \mathrm{HyperQPTL}^+$ (general case)
$\wedge^\dagger$		
$\mathrm{TeamLTL}(\varovee, \mathrm{NE}, \overset{1}{\mathsf{A}})$	$\leq$	$\exists_p \overset{u}{Q}{}^*_p \exists^*_\pi \forall_\pi \mathrm{HyperQPTL}^+$
$\wedge\!\!\mid$ [Luck 2020]		(assuming k-coherence)
$\mathrm{TeamLTL}(\sim)$	$\leq$	$\forall^k \mathrm{HyperLTL}$

Table: Expressivity results. † holds since $\mathrm{TeamLTL}(\overset{1}{\mathsf{A}}, \varovee)$ is downward closed.

- $\exists_p$ is a quantification of a new proposition
- $\overset{u}{Q}{}^*_p$ is quantification of new uniform propositions (unique value for each time step)
- $\forall_\pi$ is a quantification of a trace variable

Modes of asynchronicity

- Synchronous TeamLTL:
  - $(T, i) \models \varphi$
  - Collection of traces T with one global clock i.
- Asynchronous TeamLTL:
  - $(T, f) \models \varphi$
  - Collection of traces T with a collection of local clocks $f \colon T \to \mathbb{N}$.
  - Local clocks are completely independent.

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- TeamLTL with time evaluation functions (tefs):
  - $(T, \tau) \models \varphi$
  - Collection of traces T and a tef $\tau \colon \mathbb{N} \times T \to \mathbb{N}$ relating a global clock to local clocks.
  - The behaviour of local clocks is determined by a tef.
  - Synchronous TeamLTL is an instance, where the tef is synchronous!
  - (cf. trajectories of Bonakdarpour, Prabhakar, Sánchez, NASA Formal Methods 2020)

Complexity results for logics with tefs

Model Checking Problem for	Complexity
$\exists\mathrm{TeamLTL}(\otimes, \subseteq)$	Σ^0_1-hard
$\forall\mathrm{TeamLTL}(\otimes, \subseteq, \mathrm{NE})$	Σ^0_1-hard
$\exists$TeamCTL$^*(\otimes, \subseteq)$	Σ^0_1-hard
$\forall\mathrm{TeamCTL}(\otimes, \subseteq)$	Σ^0_1-hard
$\exists$TeamCTL$^*(\otimes)$	Σ^1_1-hard
$\mathrm{TeamCTL}^*(\mathcal{S}, \mathrm{ALL})$ for k-synchronous or k-context-bounded tefs	decidable
$\mathrm{TeamCTL}^*(\mathcal{S})$ for k-synchronous or k-context-bounded tefs, where k and the number of traces is fixed	polynomial time

Table: Complexity results overview. The Σ^0_1-hardness results follow via embeddings of synchronous $\mathrm{TeamLTL}$, whereas the Σ^1_1-hardness truly relies on asynchronity. ALL is the set of all generalised atoms and $\mathcal{S} = \{\otimes, \mathrm{NE}, \overset{1}{\mathrm{A}}, \mathrm{dep}, \subseteq\}$.

Asynchronous TeamLTL: sets and multisets

- Synchronous semantics: $(T, i) \models \varphi$ (set of traces and global clock $i \in \mathbb{N}$.)
  No difference whether T is a set or a multiset.
- Asynchronous semantics: $(T, f) \models \varphi$ (set of traces and local clocks $f : T \mapsto \mathbb{N}$.)
  A big difference whether T is a set or a multiset.

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  - $(T, f) \models F\varphi$ if there is a way to advance asynchronously on the traces such that φ becomes true.
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- Note that there is no difference in synchronous semantics (until one starts to consider quantitative atoms such as probabilistic independence).

Expressive power of asynchronous set TeamLTL

Asynchronous set TeamLTL is expressibly weak:

- TeamLTL with downward closed atomic hyperproperties is as expressive as the closure of $\forall$HyperLTL with $\wedge$ and $\vee$.
  (TeamLTL $\Rightarrow$ BC($\forall$HyperLTL) yields exponentially many small disjuncts.)
- A natural fragment of TeamLTL with all atomic hyperproperties is as expressive as the closure of $\forall$HyperLTL with $\wedge$, $\vee$, and $\neg$.

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The logics are decidable, but...

- TeamLTL($\owedge$) satisfiability and model checking are PSPACE-complete.
- For a natural fragment of TeamLTL($\owedge, \sim$) satisfiability and model checking are in TOWER(poly).

Questions

- What do you mean by asynchronicity?
  Who controls asynchronicity user vs nature vs policy?
- What are interesting hyperproperties that rely on checking unbounded number of traces in concert?
  Example: Bounded termination "F term" in synchronous TeamLTL.
- Is there some practical interest in checking whether a data is (can be seen as) decomposed from two datasets satisfying some properties?
  Example: $\mathrm{dep}(i_1, \ldots, i_n; o_1, \ldots, o_n) \vee \mathrm{dep}(i'_1, \ldots, i'_n; o_1, \ldots, o_n)$.
  The observable output $o_1, \ldots, o_n$ has two sources that determine it.

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